Math 115

Spring 2019

Lecture 16

$? \; a^2 + b^2 = c^2 \; ?$

$y = mx + b \quad ? \quad d = rt$

Ch.4 Polynomial Expression & Exponential Rules

Mathematical Expressions

It is combination of numbers, variables, and operations. NO = Sign.

$2x^2 - 5x$, $\frac{x^2-9}{x^2-4}$, $\sqrt{x^2-y^2}$, $\frac{-b \pm \sqrt{b^2-4ac}}{2a}$

1) we can evaluate M.E.

2) we can Simplify M.E.

Evaluate $-2x^2+8x$ for $x=-3$.

$$-2x^2+8x = -2(-3)^2+8(-3)$$

$$= -2\cdot 9 + 8(-3)$$

$$= -27 + (-24)$$

$$= \boxed{-51}$$

Given $x_1=2$, $y_1=-8$, $x_2=-3$, and $y_2=4$

Evaluate $\sqrt{(x_1-x_2)^2+(y_1-y_2)^2} = \sqrt{(2--3)^2+(-8-4)^2}$

$$= \sqrt{(2+3)^2+(-8-4)^2}$$

$$= \sqrt{5^2+(-12)^2} = \sqrt{25+144}$$

$$= \sqrt{169} = \boxed{13}$$

Evaluate $\dfrac{-b-\sqrt{b^2-4ac}}{2a}$ for

$a=2$, $b=5$, and $c=-7$.

$$\frac{-b-\sqrt{b^2-4ac}}{2a} = \frac{-5-\sqrt{5^2-4(2)(-7)}}{2(2)}$$

$$= \frac{-5-\sqrt{25-4(2)(-7)}}{4}$$

$$= \frac{-5-\sqrt{25+56}}{4} = \frac{-5-\sqrt{81}}{4}$$

$$= \frac{-5-9}{4} = \frac{-14}{4} = \boxed{\frac{-7}{2}}$$

Simplify:

$5x^3 - 6x^2 + 14x - 8 - 2x^3 + 5x^2 - 10x + 8$

$= 3x^3 + 4x$

Simplify:

$3(2x^2 - 5x + 4) - 2(3x^2 - 7x + 6)$

$= 6x^2 - 15x + 12 - 6x^2 + 14x - 12$

$= -1x = -x$

Simplify:

$4(x^2 - 2xy - 3y^2) - 2(2x^2 + 5xy - 6y^2)$

$= 4x^2 - 8xy - 12y^2 - 4x^2 - 10xy + 12y^2$

$= -18xy$

Exponential Rules:

① $x^n = \underbrace{x \cdot x \cdot x \cdot \cdots \cdot x}_{n \text{ times}}$

$x \rightarrow$ base

$n \rightarrow$ Exponent (Power)

$4^3 = 4 \cdot 4 \cdot 4$

$(-3)^4 = (-3)\cdot(-3)(-3)(-3)$

$\left(\frac{2}{5}\right)^2 = \left(\frac{2}{5}\right)\cdot\left(\frac{2}{5}\right)$

$(-5x)^7 =$

$\underbrace{(-5x)(-5x)(-5x)\cdots(-5x)}_{7 \text{ times}}$

$(x^2y^3)^6 = \underbrace{(x^2y^3)(x^2y^3)\cdots(x^2y^3)}_{6 \text{ times}}$

② $x^1 = x$

$7^1 = 7$

$(-8)^1 = -8$

$\left(\frac{3}{5}\right)^1 = \frac{3}{5}$

$(-7xy)^1 = -7xy$

$(4x^2 - 3y^3)^1 = 4x^2 - 3y^3$

$(-8x^2y^5)^1 = -8x^2y^5$

③ $x^0 = 1, \quad x \neq 0$

$4^0 = 1$

$(-25)^0 = 1$

$-25^0 = -1$

$2019^0 = 1$

$(2x^5)^0 = 1, \quad x \neq 0$

$(3x^2 + 8)^0 = 1$

0^0 indeterminate

④ $x^m \cdot x^n = x^{m+n}$

$x^3 \cdot x^5 = x^{3+5} = \boxed{x^8}$

$x^4 \cdot x^6 \cdot x^1 = x^{4+6+1} = \boxed{x^{11}}$

$(2xy^3)^7 \cdot (2xy^3)^3 = (2xy^3)^{7+3} = (2xy^3)^{10}$

$\left(\frac{-3x^2}{5y^6}\right)^4 \cdot \left(\frac{-3x^2}{5y^6}\right)^{12} = \boxed{\left(\frac{-3x^2}{5y^6}\right)^{16}}$

⑤ $(x^m)^n = x^{m \cdot n}$

$(x^4)^3 = x^{4 \cdot 3} = \boxed{x^{12}}$

$(x^7)^5 \cdot x^{10} = x^{35} \cdot x^{10} = \boxed{x^{45}}$

$(x^8)^2 \cdot (x^2)^7 = x^{16} \cdot x^{14}$

$= x^{16+14} = \boxed{x^{30}}$

⑥ $(xy)^n = x^n y^n$

$(2x)^5 = 2^5 x^5 = \boxed{32x^5}$

$(-3x^4)^3 = (-3)^3 (x^4)^3 = \boxed{-27x^{12}}$

$(-2x^5)^4 \cdot (5x^3)^2 = (-2)^4 (x^5)^4 (5)^2 (x^3)^2$

$= 16\, x^{20} \cdot 25\, x^6$

$= 16 \cdot 25\, x^{20} x^6 = \boxed{400x^{26}}$

① Expand: $(-4x^2)^3 = (-4x^2)\cdot(-4x^2)\cdot(-4x^2)$

② Simplify: $\left(\frac{2}{3}\right)^1 - \left(\frac{1}{5}\right)^0 = \frac{2}{3} - 1 = \frac{2}{3} - \frac{3}{3}$

$= \frac{2-3}{3}$

$= \boxed{\frac{-1}{3}}$

③ Simplify: $(x^6)^8 \cdot (x^2)^1$

$= x^{6\cdot 8} \cdot x^{2\cdot 1}$

$= x^{48} \cdot x^2 = x^{48+2} = \boxed{x^{50}}$

④ Simplify: $(-4x^2)^3 = (-4)^3(x^2)^3$

$= \boxed{-64x^6}$

⑤ Simplify: $(-10x^4)^3 \cdot (x^{-2})^6 \quad ; x \neq 0$

$= (-10)^3(x^4)^3 \cdot (x^{-2})^6$

$= -1000\, x^{12} \cdot x^{-12}$

$= -1000\, x^{12+(-12)}$

$= -1000\, x^0$

$= -1000 \cdot 1$

$= \boxed{-1000}$

⑦ $\dfrac{x^m}{x^n} = x^{m-n}$

$\dfrac{x^7}{x^3} = x^{7-3} = \boxed{x^4}$

$\dfrac{(x^4)^5}{(x^3)^4} = \dfrac{x^{20}}{x^{12}} = x^{20-12} = \boxed{x^8}$

$\dfrac{(2x^6)^4}{(4x^{12})^2} = \dfrac{2^4(x^6)^4}{4^2(x^{12})^2} = \dfrac{16x^{24}}{16x^{24}} = \boxed{1}$

⑧ $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

$\left(\frac{2}{3}\right)^4 = \frac{2^4}{3^4} = \boxed{\frac{16}{81}}$

$\left(\frac{x^6}{y^3}\right)^2 = \frac{(x^6)^2}{(y^3)^2} = \frac{x^{12}}{y^6}$

$\left(\frac{-3x^5}{4y^6}\right)^3 = \frac{(-3)^3(x^5)^3}{4^3(y^6)^3} = \boxed{\frac{-27x^{15}}{64y^{18}}}$

⑨ $x^{-n} = \frac{1}{x^n}$

$x^{-5} = \frac{1}{x^5}$

$e^{-1} = \frac{1}{e^1} = \boxed{\frac{1}{e}}$

$(x^6)^{-2} = x^{-12} = \frac{1}{x^{12}}$

$(x^5)^{-3} \cdot (x^{-2})^5$

$= x^{-15} \cdot x^{-10}$

$= x^{-25} = \boxed{\frac{1}{x^{25}}}$

Simplify

$$(-2x^4y^{-3})^5 = (-2)^5(x^4)^5(y^{-3})^5$$

$$= -32\,x^{20}\,y^{-15}$$

$$= \boxed{\frac{-32x^{20}}{y^{15}}}$$

Exponential Rules:

1) $x^n = \underbrace{x \cdot x \cdot x \cdot \cdots \cdot x}_{n \text{ times}}$

2) $x^1 = x$

3) $x^0 = 1 \; ; \; x \neq 0$

4) $x^m \cdot x^n = x^{m+n}$

5) $(x^m)^n = x^{m \cdot n}$

6) $(xy)^n = x^n y^n$

7) $\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}$

8) $\frac{x^m}{x^n} = x^{m-n}$

9) $x^{-n} = \frac{1}{x^n}$

10) $\frac{x^{-m}}{y^{-n}} = \frac{y^n}{x^m}$

11) $\left(\frac{x}{y}\right)^{-n} = \left(\frac{y}{x}\right)^n$

Simplify

1) $\dfrac{x^{-4}\,y^{7}}{x^{6}\,y^{-8}} = \dfrac{y^{7}\,y^{8}}{x^{6}\,x^{4}} = \boxed{\dfrac{y^{15}}{x^{10}}}$

2) $\left(\dfrac{2x^{6}}{3y^{5}}\right)^{-3} = \left(\dfrac{3y^{5}}{2x^{6}}\right)^{3} = \dfrac{3^{3}(y^{5})^{3}}{2^{3}(x^{6})^{3}}$

$= \boxed{\dfrac{27\,y^{15}}{8\,x^{18}}}$

Simplify:

$\left(\dfrac{-2x^{-4}}{y^{-5}}\right)^{-4}$

Hint: Work inside of ( ), take care of — exponent.

$= \left(\dfrac{-2y^{5}}{x^{4}}\right)^{-4} = \left(\dfrac{x^{4}}{-2y^{5}}\right)^{4} = \dfrac{(x^{4})^{4}}{(-2)^{4}(y^{5})^{4}}$

Work on SG 12 for Tuesday. It will be collected.

$= \boxed{\dfrac{x^{16}}{16\,y^{20}}}$